

**Closed-loop adaptive control of extreme events in a turbulent flow**Mohammad Farazmand <sup>\*</sup>*Department of Mathematics, North Carolina State University, 2311 Stinson Drive, Raleigh, North Carolina 27695-8205, USA*

Themistoklis P. Sapsis

*Department of Mechanical Engineering, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139-4307, USA*

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Extreme events that arise spontaneously in chaotic dynamical systems often have an adverse impact on the system or the surrounding environment. As such, their mitigation is highly desirable. Here, we introduce a control strategy for mitigating extreme events in a turbulent shear flow. The controller combines a probabilistic prediction of the extreme events with a deterministic actuator. The predictions are used to actuate the controller only when an extreme event is imminent. When actuated, the controller only acts on the degrees of freedom that are involved in the formation of the extreme events, exerting minimal interference with the flow dynamics. As a result, the attractors of the controlled and uncontrolled systems share the same chaotic core (containing the nonextreme events) and only differ in the tail of their distributions. We propose that such adaptive low-dimensional controllers should be used to mitigate extreme events in general chaotic dynamical systems, beyond the shear flow considered here.

DOI: [10.1103/PhysRevE.100.033110](https://doi.org/10.1103/PhysRevE.100.033110)**I. INTRODUCTION**

Many chaotic dynamical systems exhibit spontaneous extreme events which cause abrupt changes in the state of the system [1–3]. Well-known examples include extreme weather patterns, oceanic rogue waves, earthquakes, and shocks in power grids. Since extreme events cause adverse humanitarian, environmental, and financial impacts, their mitigation is of great interest.

In order to design control strategies that mitigate the extreme events, it is crucial to understand the mechanisms that generate them. The controller should either disrupt these mechanisms or counteract their effects.

Recent studies show that, in many systems, only a few degrees of freedom contribute to the formation of extreme events, even though the system as a whole may be very high dimensional [4–9]. This raises the prospect of designing simple low-dimensional controllers that mitigate the extreme events by only acting on these few degrees of freedom.

Here, we explore the feasibility of such simple controllers. We require the control design to have two specific features: (i) Low dimensionality: The controller should only act on those degrees of freedom that are involved in the formation of extreme events. This allows for the simplest possible control design and, therefore, facilitates its practical implementation. (ii) Adaptivity: We require the control to automatically actuate only when there is a high probability of an imminent extreme event. In other words, the control is inactive most of the time. Shortly before an extreme event takes place, it becomes active,

mitigating the event. The control becomes inactive again after the extreme event episode. This requires a real-time prediction scheme for the extreme events. This adaptive property is similar to the occasional proportional feedback control [10] used to stabilize equilibria and periodic orbits, except that we use a Bayesian probabilistic prediction for the occurrence of extreme events.

These requirements distinguish our approach from classical control strategies that seek to suppress the chaotic behavior of the system altogether by stabilizing a particular equilibrium state or a periodic orbit [11–17]. Instead, our approach leaves the chaotic core of the attractor (corresponding to the nonextreme events) intact and only prunes the small portion of the attractor that corresponds to extreme events (see Fig. 1, for an illustration).

Here, we demonstrate the feasibility of such extreme event mitigations on a canonical turbulent flow: the two-dimensional Navier-Stokes equation driven by a sinusoidal body force, usually referred to as the Kolmogorov flow [18–20]. Extreme events are a common feature of moderate and high Reynolds number fluid flow regardless of the external forcing or boundary conditions [21–27]. These extreme events can be divided into two broad categories: local and global. Local extreme events correspond to unusually high velocity gradients in a subset of the fluid domain [23,25]. In contrast, global extreme events cannot be pinpointed to a localized event; instead they correspond to the space-averaged quantities of the flow [24,26].

The extreme events in Kolmogorov flow are of the global type and appear as intermittent bursts of the total energy dissipation rate. Controlling local extreme events in turbulence requires a predictive scheme that, in real time, tracks the location of the extremes in the fluid domain. As such,

<sup>\*</sup>Corresponding author: farazmand@ncsu.edu

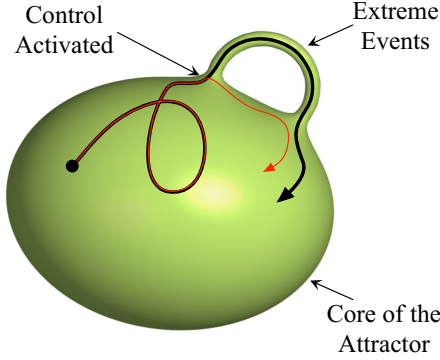


FIG. 1. Schematic of the system attractor. The thick black curve shows a trajectory of the uncontrolled system undergoing an extreme event. The thin red curve marks the trajectory of the controlled system which evades the extreme event. The black dot marks the initial state of the system.

mitigation of the local extreme events seems out of reach at the moment.

**II. PROBLEM SETUP**

We consider the incompressible Navier-Stokes equations on the two-dimensional domain  $\Omega = [0, 2\pi] \times [0, 2\pi]$  with periodic boundary conditions. Our control strategy is best described in the Fourier space. We denote the components of the velocity field by  $u_i(\mathbf{x}, t)$  ( $i = 1, 2$ ) and their Fourier transforms by  $\hat{u}_i(\mathbf{k}, t) = \int_{\Omega} u_i(\mathbf{x}, t) \exp(-i\mathbf{k} \cdot \mathbf{x}) d^2\mathbf{x} / (2\pi)^2$  where  $\mathbf{k} = (k_1, k_2) \in \mathbb{Z}^2$ ,  $\mathbf{x} = (x_1, x_2) \in \Omega$  and  $\hat{i} = \sqrt{-1}$ . The Navier-Stokes equation in the Fourier space reads [28]

$$\partial_t \hat{u}_i(\mathbf{k}, t) = -i k_m P_{ij}(\mathbf{k}) \sum_{\substack{\mathbf{p}, \mathbf{q} \in \mathbb{Z}^2 \\ \mathbf{p} + \mathbf{q} = \mathbf{k}}} \hat{u}_m(\mathbf{p}, t) \hat{u}_j(\mathbf{q}, t) - \nu |\mathbf{k}|^2 \hat{u}_i(\mathbf{k}, t) + \hat{f}_i(\mathbf{k}) + \hat{\xi}_i(\mathbf{k}, t), \quad (1)$$

where a summation over repeated indices is implied. Here,  $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j / |\mathbf{k}|^2$  denotes the Leray projection onto divergence-free vector fields where  $\delta_{ij}$  is the Kronecker  $\delta$  function. The dimensionless parameter  $\nu = \text{Re}^{-1}$  is the inverse of the Reynolds number  $\text{Re}$ . The external forcing  $\mathbf{f}(\mathbf{x}) = [\sin(k_f x_2), 0]$  is a time-independent shearing body force with the forcing wave number  $k_f = 4$ . The term  $\hat{\xi}_i(\mathbf{k}, t)$  denotes the control to be discussed shortly.

To solve system (1) numerically, we use a standard pseudospectral method with 2/3 dealiasing [29]. At the lowest Reynolds number considered here ( $\text{Re} = 40$ ), we use  $128 \times 128$  Fourier modes, whereas, at higher Reynolds numbers, we use  $256 \times 256$  modes. For the temporal integration, we use the adaptive Runge-Kutta scheme RK5(4) of Dormand and Prince [30] with relative and absolute tolerances set to  $10^{-5}$ .

**III. UNCONTROLLED SYSTEM**

Much is known about the uncontrolled system where  $\hat{\xi}_i \equiv 0$ . In particular, at Reynolds numbers  $\text{Re} > 35$ , the uncontrolled system is chaotic with sporadic bursts of the energy

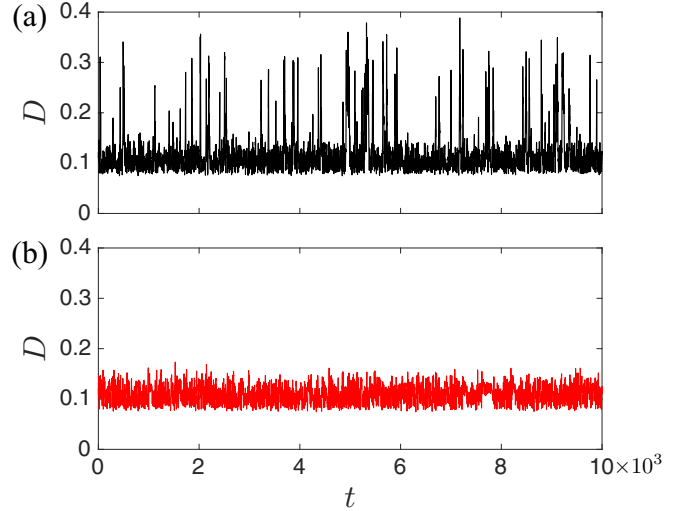


FIG. 2. The time series of the energy dissipation rate  $D$  at Reynolds number  $\text{Re} = 40$ . (a) Uncontrolled system and (b) controlled system.

dissipation rate [31],

$$D = \frac{\nu}{(2\pi)^2} \int_{\Omega} |\nabla \mathbf{u}|^2 d^2\mathbf{x} = \nu \sum_{\mathbf{k} \in \mathbb{Z}^2} |\mathbf{k}|^2 |\hat{\mathbf{u}}(\mathbf{k})|^2. \quad (2)$$

During these bursts, the energy dissipation  $D$  increases to several standard deviations above its expected value [see Fig. 2(a)]. Using a variational method, Farazmand and Sapsis [6] showed that these bursts are preceded by a nonlinear energy transfer from the Fourier mode  $\hat{\mathbf{u}}(1, 0)$  to mode  $\hat{\mathbf{u}}(0, 4)$ .

Shortly before an extreme energy dissipation event is observed, most of the energy content of the Fourier mode  $\hat{\mathbf{u}}(1, 0)$  is transferred to the Fourier mode  $\hat{\mathbf{u}}(0, 4)$ . This transfer of energy from the lower mode (1,0) to the higher mode (0,4) leads to an increase in the energy dissipation rate  $D$ . Examining the last identity in Eq. (2) reveals why such an energy transfer leads to an increase in the energy dissipation. Since higher Fourier modes are weighted by a larger prefactor  $|\mathbf{k}|^2$ , transfer of energy from any lower Fourier mode to a higher Fourier mode leads to an increase in the energy dissipation rate  $D$  (assuming that the total kinetic energy does not change significantly). In principle, any such a downscale transfer of energy will increase the energy dissipation rate. In Kolmogorov flow, however, it is the particular abrupt transfers from the lower Fourier mode  $\hat{\mathbf{u}}(1, 0)$  to the higher Fourier mode  $\hat{\mathbf{u}}(0, 4)$  that is responsible for the bursting behavior of the energy dissipation rate [6].

One can go one step further and ask: What triggers this abrupt transfer of energy? Unfortunately, the answer to this question is still unknown. Recent results on Burgers equation [32,33] suggest that the answer lies in resonances between the phases of the Fourier modes involved in the energy transfer. However, for Kolmogorov flow, this possibility remains to be investigated. Nonetheless, we show that even the available partial knowledge about the precursors to extreme events is sufficient for their prediction and suppression in the Kolmogorov flow.

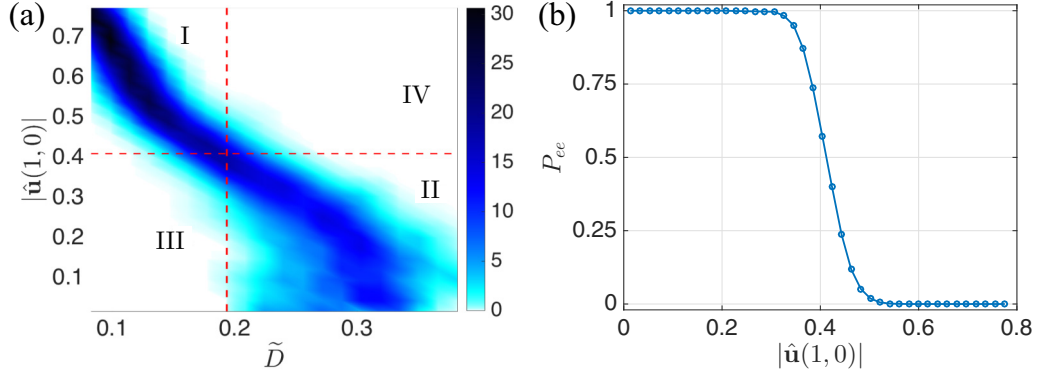


FIG. 3. Prediction of extreme events at  $\text{Re} = 40$ . (a) Conditional probability density function (PDF) of  $\tilde{D}(t) = \max_{s \in [t + \tau_p, t + \tau_p + \Delta \tau_p]} D(s)$  given as  $\hat{\mathbf{u}}(1, 0, t)$ . (b) Probability  $P_{ee}$  of an extreme dissipation event occurring over the future time interval  $[t + \tau_p, t + \tau_p + \Delta \tau_p]$  given the current value of  $|\hat{\mathbf{u}}(1, 0, t)|$ .

In particular, the energy content of mode  $\hat{\mathbf{u}}(1, 0)$  can be used as a predictive indicator for upcoming extremes of the energy dissipation rate  $D$ . To quantify the predictive skill of this indicator, we use the conditional probability of  $\tilde{D}(t)$  given  $\lambda(t) = |\hat{\mathbf{u}}(1, 0, t)|$  at a given time  $t$ , where  $\tilde{D}(t) = \max_{s \in [t + \tau_p, t + \tau_p + \Delta \tau_p]} D(s)$  is the maximum value of the energy dissipation rate  $D$  over the short future time interval  $[t + \tau_p, t + \tau_p + \Delta \tau_p]$ . The prediction time  $\tau_p$  determines how far in advance the extreme events are predicted. Here, we set  $\tau_p = \Delta \tau_p = 1.0 \simeq 2\tau_e$  which is approximately equal to two eddy turnover times,  $\tau_e = \sqrt{\nu/\mathbb{E}[D]}$ . Here,  $\mathbb{E}$  denotes the expected value.

Figure 3(a) shows the conditional probability density  $p_{\tilde{D}|\lambda} = p_{\tilde{D},\lambda}/p_\lambda$  where  $p_\lambda$  is the probability density of  $\lambda$ , and  $p_{\tilde{D},\lambda}$  is the joint probability density of  $\tilde{D}$  and  $\lambda = |\hat{\mathbf{u}}(1, 0)|$ . This conditional PDF is estimated from long term direct numerical simulations.

The vertical dashed line in Fig. 3(a) marks the threshold  $D_e$  for extreme dissipation events such that  $D > D_e$  constitutes an extreme event. Here, the threshold is set to the mean plus two standard deviations of the dissipation, i.e.,  $D_e = \mathbb{E}[D] + 2\sigma(D) \simeq 0.2$ . The horizontal dashed line marks the corresponding threshold  $\lambda_e$  for the indicator  $\lambda = |\hat{\mathbf{u}}(1, 0)|$ . These two lines divide the conditional PDF plot into four quadrants I–IV as marked in Fig. 3(a). Below, we describe the significance of each quadrant.

**Quadrant I (correct rejections):** Most of the density of the conditional probability  $p_{\tilde{D}|\lambda}$  is concentrated in this quadrant where  $|\hat{\mathbf{u}}(1, 0)| > \lambda_e$  and  $\tilde{D} < D_e$ . The relatively large values of  $|\hat{\mathbf{u}}(1, 0)|$  indicate that no significant nonlinear transfer of energy from mode  $\hat{\mathbf{u}}(1, 0)$  to mode  $\hat{\mathbf{u}}(0, 4)$  has taken place and, therefore, no upcoming extreme events are expected. Since, in this quadrant, we also have  $\tilde{D} < D_e$ , this implies that the indicator correctly predicted no upcoming extreme events.

**Quadrant II (correct predictions):** In this quadrant, we have  $|\hat{\mathbf{u}}(1, 0)| < \lambda_e$  and  $\tilde{D} > D_e$ . As mentioned earlier, prior to an extreme event,  $|\hat{\mathbf{u}}(1, 0)|$  becomes small since most of its energy is transferred to mode  $\hat{\mathbf{u}}(0, 4)$  through internal nonlinear interactions. Therefore,  $|\hat{\mathbf{u}}(1, 0)| < \lambda_e$  signals an upcoming extreme event. Since, in this quadrant, we also have  $\tilde{D} > D_e$ , the indicator has correctly predicted the upcoming occurrence of an extreme event.

**Quadrant III (false positives):** This quadrant corresponds to false positive predictions. Since  $|\hat{\mathbf{u}}(1, 0)| < \lambda_e$ , the indicator predicts an upcoming extreme event. However, we have  $\tilde{D} < D_e$  which implies no extreme events actually took place.

**Quadrant IV (false negatives):** This quadrant corresponds to false negative predictions. Since  $|\hat{\mathbf{u}}(1, 0)| > \lambda_e$ , the indicator predicts no upcoming extreme events. However, we have  $\tilde{D} > D_e$  which implies that an extreme event actually took place.

Clearly, quadrants III and IV are undesirable since the indicator incorrectly predicts the extremes or lack thereof. However, only a small portion of the conditional density  $p_{\tilde{D}|\lambda}$  resides in these quadrants, implying that  $|\hat{\mathbf{u}}(1, 0)|$  serves as a reliable indicator of extreme dissipation events. In fact, the rate of false positive and false negative predictions are 0.85% and 0.26%, respectively, that is, the overwhelming majority of extreme events are predicted correctly.

We also define the probability that an extreme dissipation event ( $D > D_e$ ) takes place over the future time interval  $[t + \tau_p, t + \tau_p + \Delta \tau_p]$  given  $\lambda = |\hat{\mathbf{u}}(1, 0, t)|$  at the current instant  $t$ . We denote this quantity by  $P_{ee}$  and refer to it as the *probability of upcoming extreme events* which is defined by taking the marginal of the conditional probability  $p_{\tilde{D}|\lambda}$ , i.e.,

$$P_{ee}(\lambda) = \int_{D_e}^{\infty} p_{\tilde{D}|\lambda}(\zeta, \lambda) d\zeta. \quad (3)$$

For a given  $\lambda = |\hat{\mathbf{u}}(1, 0, t)|$ ,  $P_{ee}(\lambda)$  measures the probability that  $D(s) > D_e$  for some time  $s \in [t + \tau_p, t + \tau_p + \Delta \tau_p]$ .

Figure 3(b) shows the probability of upcoming extreme dissipation events for the Kolmogorov flow. For relatively large values of  $|\hat{\mathbf{u}}(1, 0)|$ , the probability of upcoming extremes is virtually zero. As mode  $\hat{\mathbf{u}}(1, 0)$  transfers its energy to mode  $\hat{\mathbf{u}}(0, 4)$  and, therefore,  $|\hat{\mathbf{u}}(1, 0)|$  becomes relatively small, the probability of upcoming extremes approaches one, signaling the high likelihood of an upcoming extreme event. Below, we use the predictions obtained by  $P_{ee}$  to decide whether or not to actuate the control.

#### IV. CONTROLLED SYSTEM

Recall that the extreme energy dissipation events are instigated by a nonlinear transfer of energy from mode  $\hat{\mathbf{u}}(1, 0)$  to

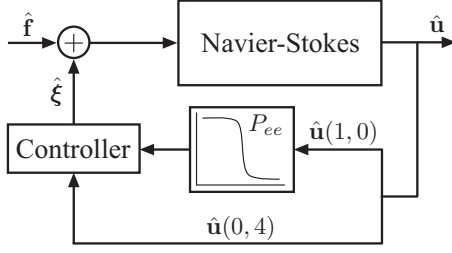


FIG. 4. The block diagram of the control strategy. The Fourier transforms of the external forcing, the velocity field, and the control term are denoted by  $\hat{\mathbf{f}}$ ,  $\hat{\mathbf{u}}$ , and  $\hat{\boldsymbol{\xi}}$ , respectively. The Fourier mode  $\hat{\mathbf{u}}(1,0)$  is used to measure the probability of upcoming extreme events  $P_{ee}$ . The control is proportional to the Fourier mode  $\hat{\mathbf{u}}(0,4)$  where coincidentally  $\mathbf{k}_f = (0,4)$  is the wave number of the external forcing.

mode  $\hat{\mathbf{u}}(0,4)$ . Therefore, it is natural to attempt to mitigate these extreme events by removing the excess energy from mode  $\hat{\mathbf{u}}(0,4)$ . We accomplish this by designing the control term  $\boldsymbol{\xi}$  to have the form of a damping on mode  $\hat{\mathbf{u}}(0,4)$ . To this end, we set  $\hat{\xi}_i(\mathbf{k}, t) \propto -[\hat{u}_i(\mathbf{k}_f, t)\delta_{\mathbf{k}, \mathbf{k}_f} + \overline{\hat{u}_i(\mathbf{k}_f, t)}\delta_{\mathbf{k}, -\mathbf{k}_f}]$  where  $\mathbf{k}_f = (0,4)$ . The complex conjugate term acting on the wave number  $-\mathbf{k}_f$  is necessary to ensure that the resulting velocity field  $\mathbf{u}(\mathbf{x}, t)$  is real valued.

Note that this control only acts on the Fourier mode  $\hat{\mathbf{u}}(\mathbf{k}_f)$  [and its complex conjugate counterpart  $\hat{\mathbf{u}}(-\mathbf{k}_f)$ ]. Examining Eq. (1) and neglecting the Navier-Stokes dynamics for the moment, the controller acts on this mode as  $\partial_t \hat{\mathbf{u}}(\mathbf{k}_f, t) \propto -\hat{\mathbf{u}}(\mathbf{k}_f, t)$  which damps the excess energy content of the mode exponentially fast  $|\hat{\mathbf{u}}(\mathbf{k}_f, t)| \propto e^{-t}$ .

We also would like the control to be actuated only when an extreme event is about to take place. To this end, we define

$$\hat{\xi}_i(\mathbf{k}, t) = -\frac{1}{\tau_c} P_{ee}(t) [\hat{u}_i(\mathbf{k}_f, t)\delta_{\mathbf{k}, \mathbf{k}_f} + \overline{\hat{u}_i(\mathbf{k}_f, t)}\delta_{\mathbf{k}, -\mathbf{k}_f}], \quad (4)$$

where  $P_{ee}(t)$  is shorthand for  $P_{ee}[|\hat{\mathbf{u}}(1,0,t)|]$  [see Fig. 3(b)]. When the probability of upcoming extreme events is zero, the control is inactive since  $P_{ee} = 0$ . However, as that probability

increases, the control term becomes active gradually until  $P_{ee}$  approaches one, and the controller becomes fully active. After the extreme event episode, the probability  $P_{ee}$  decays back to zero, and consequently, the controller turns off. The parameter  $\tau_c$  is the time lag between the control becoming fully active ( $P_{ee} = 1$ ) and the turbulent velocity field responding to the action of the control. Here, we set  $\tau_c = \tau_p = 1.0$ . Figure 4 summarizes the control strategy in a block diagram.

Taking the inverse Fourier transform, the control can be written in the physical space as

$$\xi_i(\mathbf{x}, t) = -[2P_{ee}(t)/\tau_c]r_i(t)\cos[k_f x_2 + \phi_i(t)], \quad (5)$$

where  $r_i$  and  $\phi_i$  are the amplitude and phase of the Fourier mode  $\hat{u}_i(\mathbf{k}_f, t)$ , respectively, so that  $\hat{u}_i(\mathbf{k}_f, t) = r_i e^{i\phi_i}$ . Since the velocity field is divergence free  $\mathbf{k}_f \cdot \hat{\mathbf{u}}(\mathbf{k}_f) = 0$ , we have  $r_2(t) = 0$ . As a result,  $\xi_2(\mathbf{x}, t) \equiv 0$ , and the control only acts on the horizontal component  $u_1(\mathbf{x}, t)$  of the velocity field.

Furthermore, numerical simulations suggest that, in the uncontrolled system, the phase  $\phi_1$  oscillates around  $-\pi/2$  with a small standard deviation. For instance, at  $\text{Re} = 40$ , we have  $\sigma(\phi_1) \simeq 0.03\pi$ , where  $\sigma(\phi_1)$  denotes the standard deviation of  $\phi_1$ . As a result, the controller can be further simplified by assuming  $\phi_1 = -\pi/2$  which implies

$$\xi_1(\mathbf{x}, t) = -[2P_{ee}(t)/\tau_c]r_1(t)\sin(k_f x_2), \quad \xi_2(\mathbf{x}, t) = 0. \quad (6)$$

We note that this simplified control is a scalar multiple of the external forcing  $\mathbf{f}$ . The corresponding probability distributions of the energy dissipation  $D$  are nearly identical whether we use the full control (5) or its simplified form (6).

Figure 5 shows the closeup view of an extreme event at  $\text{Re} = 40$ ; it compares the uncontrolled and controlled system trajectories starting from the same initial condition. Initially, the probability of upcoming extreme events is zero ( $P_{ee} = 0$ ), and therefore, the control is inactive. As a result, the trajectories of the uncontrolled and controlled systems coincide. Around time  $t \simeq 30$ , the probability  $P_{ee}$  increases towards one, the control becomes active, and the trajectory of the controlled system deviates from the uncontrolled system. Shortly after  $t = 30$ , the uncontrolled system undergoes an extreme event

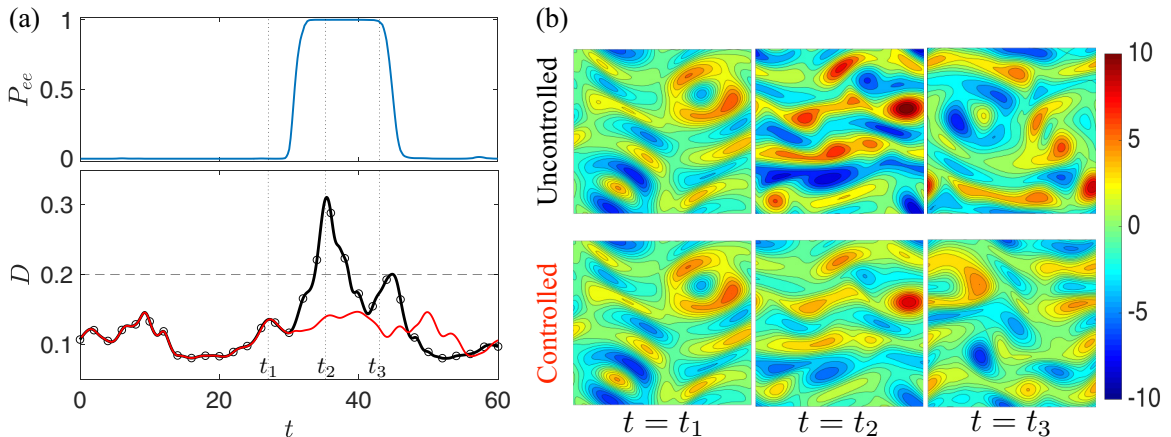


FIG. 5. Controlled versus uncontrolled systems at  $\text{Re} = 40$ . (a) Energy dissipation rate  $D$  of the uncontrolled (black circles) and controlled (solid red) systems as a function of time. The horizontal dashed line marks the threshold  $D_e = 0.2$  for the extreme events. The top panel shows the probability  $P_{ee}$  of upcoming extreme events. (b) The corresponding vorticity fields at times  $t_1 = 27.0$ ,  $t_2 = 35.2$ , and  $t_3 = 43.0$ .

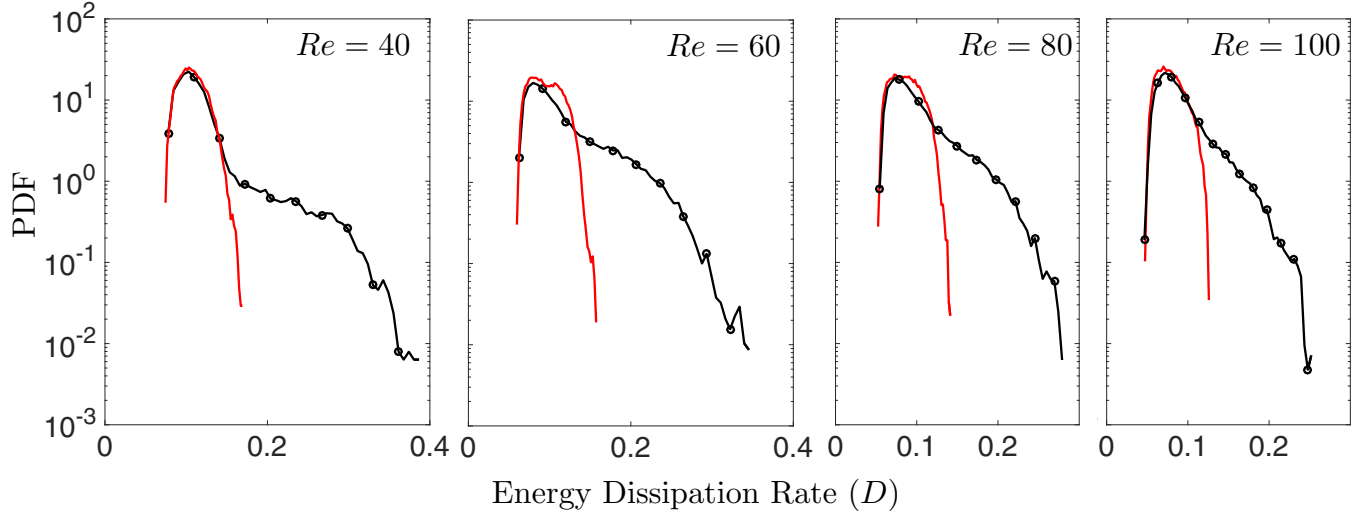


FIG. 6. The PDF of the uncontrolled (black circles) and controlled (solid red) systems at Reynolds numbers  $Re = 40, 60, 80$  and  $100$ . Each PDF is estimated from 50 000 data points.

( $D > D_e \simeq 0.2$ ). However, the controlled system successfully evades any such event, and its energy dissipation rate remains below the threshold  $D_e$ . A longer time series of the energy dissipation of the controlled system is shown in Fig. 2(b).

Figure 6 shows the PDF of the energy dissipation rate from long term simulations at Reynolds numbers  $Re = 40, 60, 80$  and  $100$ . The PDFs corresponding to the uncontrolled system have heavy tails due to the extreme dissipation events. However, the PDFs of the controlled systems have no such heavy tails, indicating the successful mitigation of extreme events. Furthermore, the core of the PDFs (corresponding to nonextreme events) are very similar for both controlled and uncontrolled systems. This implies that the controller does not fundamentally change the nature of the flow; it only mitigates the extreme events, forcing the turbulent trajectories to stay on the core of the turbulent attractor (cf. Fig. 1).

Note that, although our controller acts on mode  $\hat{\mathbf{u}}(0, 4)$ , its activation is decided based on  $P_{ee}$  which depends on the precursor  $|\hat{\mathbf{u}}(1, 0)|$ . We have also tried to actuate our controller based on the modulus of the controlled mode  $|\hat{\mathbf{u}}(0, 4)|$  instead of  $|\hat{\mathbf{u}}(1, 0)|$ . Although this control strategy suppresses some of the extreme events, it fails to remove the heavy tail events altogether.

We conclude by commenting on the possible experimental implementation of our control strategy. Kolmogorov-like flows have been studied in the laboratory experiments by electromagnetically driving a thin layer of the electrolyte [34–36]. The electromagnetic force is exerted by an array of magnets with alternating magnetization, generating the sinusoidal forcing in Eq. (1).

These laboratory experiments differ from our Kolmogorov flow in their boundary conditions and that they are not strictly two dimensional. As such, the mechanism underlying the extreme events in the laboratory experiments should be investigated based on more accurate models, such as the quasi-two-dimensional model developed in Ref. [37]. If the extreme event mechanisms turn out to be similar to our Kolmogorov model, then our results are relevant to the experiments.

Since our control only acts on the same wave number as the external forcing, actuating the control in the laboratory experiments amounts to adjusting the magnitude of the external forcing (or equivalently the magnitude of the external magnetic field). This magnitude would depend on the probability  $P_{ee}$  which, in turn, depends on the magnitude of the Fourier mode  $\hat{\mathbf{u}}(1, 0)$ . Therefore, experimental implementation of our control strategy would require high-speed velocimetry [38,39] so that the control can be actuated in time to counteract the extreme events. Since the precursor  $|\hat{\mathbf{u}}(1, 0)|$  corresponds to large scales [or equivalently, the small wave number  $\mathbf{k} = (1, 0)$ ], only a low-pass filtered measurement of the velocity is sufficient.

## V. CONCLUSIONS

A plethora of high-dimensional chaotic dynamical systems exhibits spontaneous extreme events. Recent advances in quantification and prediction of extreme events show that only a few degrees of freedom might be directly involved in the formation of these events. This raises the prospect of designing low-dimensional controllers that only act on these few degrees of freedom in order to mitigate the extreme events.

Here, we demonstrated the feasibility of such simple controllers on a turbulent fluid flow. Although acting on a single Fourier mode, our controller succeeded in suppressing all the extreme dissipation events.

We emphasize two important features of our controller: low dimensionality and adaptivity. The low dimensionality of the controller is desirable as it facilitates its practical implementation. This is feasible due to the inherent low dimensionality of the precursors to extreme events. Adaptivity refers to the fact that the controller is off most of the time and becomes active only when there is a probabilistic prediction of an imminent extreme event. Unlike classical methods for controlling chaos, our controller does not attempt to suppress chaos altogether. Instead, it only acts for relatively short

periods of time and exerts minimal interference with system dynamics. As such, the controlled system is still chaotic but contains no extreme events.

We propose that these two features (namely, low dimensionality and adaptivity) should form the basis of controlling extreme events more generally, beyond our fluid system.

## ACKNOWLEDGMENTS

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