Influence of Deck Submergence Events on Extreme Properties of Wave-Induced Vertical Bending Moment

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ABSTRACT

Extreme values of vertical bending moment may be related with large relative motions of deck points in the bow leading to submergence of a portion of the deck. Once part of the hull in the bow is submerged, its contribution to wave-induced vertical bending moment (VBM) becomes constant. This effect has been observed with numerical simulation and has been described previously.

The deck submergence effect is seen in the VBM distribution as an inflection point where the tail of the distribution turns down and becomes lighter. The position of this inflection point has some practical importance. If the return period corresponding to the inflection point is less than the service life, the deck submergence effect has an influence on the lifetime VBM.

Keywords: *Wave-induced VBM, Extreme values, Deck submergence*

1. INTRODUCTION

The problem of evaluation of extreme values of vertical bending moment (VBM) over the lifetime of a ship is introduced. A violation of hull integrity due to exceedance of global load limit is a major safety hazard along with loss of stability. Most of these safety hazards do have a stochastic nature. A probabilistic description of VBM was developed shortly after introduction of irregular waves into naval architecture practice by St. Denis and Pierson (1953). The VBM at a given section of a ship hill was considered as a normal stationary process, e.g. Salvesen et al. (1970). The application of extreme value theory (e.g. Coles, 2001) to the VBM process was introduced by Ochi and Wang (1976) and further developed by Ochi (1981). The departure from linear assumptions led to the development of time-domain numerical simulation approaches (e.g. Shin at al. 2003). The extreme values of VBM are estimated by fitting a Weibull distribution to the results of numerical simulation. Essentially, the transition to time-domain has turned the problem of VBM extreme values evaluation into an extrapolation problem.

The extrapolation of a response of a nonlinear system is a formidable task. Nonlinearity of ship motion and loads manifest itself as a change of the physics of the problem during large ship motions due to a significant variation of the submerged portion of ship hull. This nonlinearity may lead to significant uncertainty of an extrapolated estimate. The way to control this uncertainty is to include some physical consideration into statistical model (e.g. Weems et al., 2019) by considering a reduced-order qualitatively correct model, where analytical or semi-analytical solution for extremes may be derived (Belenky et al. 2019).

Sapsis et al. (2020) proposed an approximate volume-based model for VBM by expanding station areas with Taylor series and keeping the second order terms. A main linear assumption of vertical sidewall was abandoned and a station slope at the waterline was introduced. Brown and Pipiras (2020) derived a semi-analytical probability density function (PDF) for the approximate volume based model. Accounting for the station slope at the waterline explains the asymmetry of the PDF observed in numerical simulations.

Further study of the extreme value properties of VBM is described with the approximate volume-based model by considering additional factors.

2. DISTRIBUTION OF WAVE-INDUCED VBM

2.1 Approximate Volume-Based Model

Following (Sapsis et al., 2020), the wave model has to be simplified significantly in order to facilitate a semi-analytical solution for PDF. The wave is monochromatic in space, while amplitudes are stochastic processes in time. The model is somewhat similar to Grim (1961) effective wave, but it is a running wave, not a stationary wave. The wave elevation ζ_w is expressed as:

$$\zeta_w(x,t) = a_c(t)\cos(k_w x - \omega_w t) + a_s(t)\sin(k_w x - \omega_w t), \quad (1)$$

where x is a spatial coordinate in the direction of propagation of the wave (the wave is longitudinal), a_c and a_s are independent, identically distributed normal stochastic processes, but may have auto-correlation, and k_w and ω_w are wave number and wave frequency. The dispersion relation is taken for Airy waves:

$$k_w = \frac{\omega_w^2}{g}; \;\; k_w = \frac{2\pi}{\lambda_w} \quad (2)$$

The wave length λ_w is meant to be close, but not necessarily exactly equal, to ship length, while g is gravity acceleration.

The problem is considered in longitudinal waves, so ship motions are limited to heave ζ_g and pitch θ, diffraction forces are not yet included:

$$\begin{aligned}(m + A_{33})\ddot{\zeta}_g + B_{33}\dot{\zeta}_g + A_{35}\ddot{\theta} + B_{35}\dot{\theta} \\ + F_{FKHS}(\zeta_g, \theta, \zeta_w) = 0, \\ (I_y + A_{55})\ddot{\theta} + B_{55}\dot{\theta} + A_{53}\ddot{\zeta}_g + B_{53}\dot{\zeta}_g \\ + M_{FKHS}(\zeta_g, \theta, \zeta_w) = 0,\end{aligned} \quad (3)$$

where m is the ship mass, I_y is the mass moment of inertia relative to transverse axis y, A_{nn} and B_{nn} stand for hydrodynamic coefficients of added mass and damping (nk is a motion index following standard hydrodynamic notion, 3 is heave, and 5 is pitch). The index '*FKHS*' means "Froude-Krylov and Hydrostatic", while symbol F identifies vertical force, symbol M means a moment of the vertical force about the transversal axis y.

The wave-induced vertical bending moment for the section ψ (relative to the midship) is defined as:

$$M_\psi(t) = \int_{\psi}^{L/2} \left(\sum dF(x)\right)x = \int_{-L/2}^{\psi} \left(\sum dF(x)\right)x \quad (4)$$

where $\sum dF(x)$ is a sum of all the forces acting on a section of the hull between x and $x + dx$. The equality between the integrals in equation (4) is a result of including all the forces in the sum $\sum dF(x)$, *i.e.* a condition of dynamic equilibrium, essentially expressed by the system of differential equations (3).

The distribution of heave, pitch and their derivatives is assumed approximately normal within the range of interest. This assumption is not equivalent to a linear assumption as the variance of heave and pitch can be estimated while accounting for nonlinearity. Nonlinearity will eventually affect the tail of the pitch distribution, as pitch is somewhat similar to roll. Heave may be expected to have a light tail as well since a ship cannot leave the fluid domain completely, neither can she sink because of motions in waves. This assumption simply states that nonlinearity will affect VBM distribution long before it will affect the heave and pitch distributions.

The assumption of normality for damping and inertial forces acting on a section may be a bit stronger than those on heave and pitch, but generally follows the same logic. Normally distributed components of the integrals in equation (4) cannot influence the VBM distribution beyond variance (inertia and damping have zero mean). Weight is constant and affects the mean value only. Thus, the shape of the VBM distribution is defined by nonlinearity of the Froude-Krylov and hydrostatic forces. These set of assumptions is also justified by the results of Sapsis et al. (2020) where the PDF, based on these assumptions, was successfully scaled to match a histogram from a large-size sample.

The Froude-Krylov and hydrostatic force acting on a section between x and $x + dx$ is expressed as:

$$dF_{FKHS} = \rho g A(x,z)dx = \rho g A\left(x, \theta x + \zeta_g - \zeta_w\right)dx \tag{5}$$

where ρ is the density of water, ζ_w is local wave elevation and $A(x, z)$ is an area of a station at longitudinal location x, submerged by the value $z = x + \zeta_g - \zeta_w$.

Following Sapsis et al. (2020), the area of a station is expanded into a Taylor series up to the Q (the superscript in parenthesis means derivative):

$$A\left(x, \theta x + \zeta_g - \zeta_w\right) = A\left(x, \theta x - \zeta_g\right) + \sum_{q=1}^{Q} \frac{A^{(q)}(x, \theta x - \zeta_g)}{q!} \zeta_w^q \tag{6}$$

The first-order expansion with equation (6) is equivalent to the assumption that ship sides are vertical, and the first derivative of the station area is a local beam. This assumption is the basis of the linear theory (*e.g.* Salvesen et al., 1970). The second-order expansion reflects slope of a station at the waterline, *i.e.* accounts for asymmetry of a hull relative to the waterplane. Sapsis et al. (2020) show that inclusion of the second order term in equation (6) makes the model to reproduce observed asymmetry between sagging and hogging.

The shape of the VBM PDF is defined by the nonlinearity of the Froude-Krylov and hydrostatic forces. The influence of inertia and damping affect location and scale only and can be added later as an aid for the fitting process. Heave and pitch motion may affect the shape of the PDF through the expansion (6). Sapsis et al. (2020) accounted for pitch through averaging the coefficients of the expansion (6) over the normal PDF of pitch. The next logical step is to add averaging over heave.

Neglecting inertia and damping, the VBM at time $t = 0$ for particular values of heave and pitch is expressed as

$$\begin{aligned} M_\psi\left(t, \zeta_g, \theta\right) &= \int_\psi^{\frac{L}{2}} x dF_{FKHS} = \\ &m_{\psi 0}\left(\zeta_g, \theta\right) + \\ &a_c(t) m_{\psi c1}\left(\zeta_g, \theta\right) + a_s(t) m_{\psi s1}\left(\zeta_g, \theta\right) + \\ &0.5 a_c^2(t) m_{\psi c2}\left(\zeta_g, \theta\right) + \\ &0.5 a_s^2(t) m_{\psi c2}\left(\zeta_g, \theta\right) + \\ &a_c(t) a_s(t) m_{\psi cs}\left(\zeta_g, \theta\right) \end{aligned} \tag{7}$$

The functions m in equation (7) are expressed as follows (dropping arguments ζ_g, θ from the formulae for brevity):

$$\begin{aligned} m_{\psi 0} &= \int_\Psi^{\frac{L}{2}} x A\left(x, \theta x - \zeta_g\right) dx \\ m_{\psi c1} &= \int_\Psi^{\frac{L}{2}} x A'\left(x, \theta x - \zeta_g\right) \cos(k_w x)\, dx \\ m_{\psi s1} &= \int_\Psi^{\frac{L}{2}} x A'\left(x, \theta x - \zeta_g\right) \sin(k_w x)\, dx \\ m_{\psi c2} &= \int_\Psi^{\frac{L}{2}} x A''\left(x, \theta x - \zeta_g\right) \cos(k_w x)\, dx \\ m_{\psi s2} &= \int_\Psi^{\frac{L}{2}} x A''\left(x, \theta x - \zeta_g\right) \sin(k_w x)\, dx \\ m_{\psi cs} &= \\ &\int_\Psi^{\frac{L}{2}} x A''\left(x, \theta x - \zeta_g\right) \cos(k_w x) \sin(k_w x)\, dx \end{aligned} \tag{8}$$

Similar to Sapsis et al. (2020), the coefficients in equations (8) are averaged over a reasonable range (say, four standard deviation of heave and pitch), *e.g.*:

$$\mu_{\psi c1} = \int_{-2\sigma_\theta}^{2\sigma_\theta} \int_{-2\sigma_\zeta}^{2\sigma_\zeta} m_{\psi c1}(\zeta_g, \theta) f(\zeta_g, \theta) d\zeta_g d\theta \quad (9)$$

where f stands for normal bivariate PDF with parameters estimated from time-domain simulations. Finally, the approximate volume-based model is expressed as

$$\begin{aligned} M_\psi(t) = {} & \mu_{\psi 0} + \\ & a_c(t)\mu_{\psi c1} + a_s(t)\mu_{\psi s1} + \\ & 0.5a_c^2(t)\mu_{\psi c2} + 0.5a_s^2(t)\mu_{\psi c2} + \\ & a_c(t)a_s(t)\mu_{\psi cs} \end{aligned} \quad (10)$$

where functions μ_ψ are defined analogously to equation (9).

The model (10) includes some incremental additions compared to Sapsis et al. (2020): calm-water value of VBM and the influence of pitch. Nevertheless, it still cannot be used directly, as fitting to real data is required by adjusting standard deviation of a_c and a_s.

2.2 Semi-Analytical PDF: Case 1

Brown and Pipiras (2020) derived semi-analytical formula for PDF of the vertical bending moment:

$$f(M_\psi) = \frac{1}{2}\int_0^{2\pi} f\left(\cos\varphi\sqrt{M_\psi - M_S}, \sin\varphi\sqrt{M_\psi - M_S}\right) d\varphi \quad (11)$$

where f stands for bivariate normal, M_ψ is defined by formula (10), while

$$M_S = \frac{a_2^2 b_1 + a_1^2 b_2 - a_1 a_2 b_{12}}{4b_1 b_2 - b_{12}^2} \quad (12)$$

where

$$\begin{aligned} & a_1 = \mu_{mc}\sigma_a \; ; \; a_2 = \mu_{ms}\sigma_a \\ & b_1 = -0.5\mu_{mc2}\sigma_a^2 \;\; b_2 = -0.5\mu_{ms2}\sigma_a^2 \\ & b_{12} = -\mu_{mcs}\sigma_a^2 \end{aligned} \quad (13)$$

where σ_a is a standard deviation of random amplitudes a_c and a_s. The formula (11) can be applied when:

$$4b_1 b_2 - b_{12}^2 > 0 \quad (14)$$

Parameters of bivariate distribution f in the equation (11) are two mean values E_1 and E_2, two variances V_1 and V_2, and a correlation coefficient r_{12}, given as:

$$\begin{aligned} & E_1 = -\frac{a_2}{2\sqrt{b_2}} \; ; \; E_2 = \frac{a_2 b_{12} - 2a_1 b_2}{2\sqrt{b_2}\sqrt{4b_1 b_2 - b_{12}^2}} \\ & V_1 = \frac{b_{12}^2}{4b_2} - b_2 \; ; V_2 = \frac{4b_1 b_2 - b_{12}^2}{4b_2} \\ & r_{12} = \frac{b_{12}\sqrt{4b_1 b_2 - b_{12}^2}}{2b_2\sqrt{V_1 V_2}} \end{aligned} \quad (15)$$

2.3 Semi-Analytical PDF: Case 2

If the condition (14) is not satisfied, the PDF of VBM is expressed as:

$$\begin{aligned} f(M_\psi) = {} & \int_0^\infty \frac{1}{2u} f(M_1, M_2) du + \\ & \int_0^\infty \frac{1}{2u} f(M_3, M_4) du \end{aligned} \quad (16)$$

where

$$\begin{aligned} & M_1 = \frac{1}{2}\left(\frac{M_\psi + M_S}{u} + u\right) \\ & M_2 = \frac{1}{2}\left(\frac{M_\psi + M_S}{u} - u\right) \\ & M_3 = -\frac{1}{2}\left(\frac{M_\psi + M_S}{u} + u\right) \\ & M_4 = -\frac{1}{2}\left(\frac{M_\psi + M_S}{u} - u\right) \end{aligned} \quad (17)$$

Derivation of the PDF in the case 2 is analogous to the case 1 described in Brown and Pipiras (2020). The main difference between the cases is that the case 1 PDF has a limit in hogging, while the case 2 does not. The hogging limit of the case 1, whose value lays well outside of the practical range, is believed to be an artefact of the squares used in the model (10).

2.4 Probability Density Functions

The PDFs (11) and (16) are derived from the approximate volume-based model (10) that describes VBM, created only by the Froude-Krylov and hydrostatic (FKHS) forces. While the FKHS forces represent the main nonlinearity of the problem, the model (10) does not yet describe the complete problem as the inertia and damping forces are not included. Thus the PDFs (11, 16) have to be treated as a shape of the actual VBM distribution, and need to be scaled with σ_a as a shape parameter. As an incremental improvement from Sapsis et al. (2020), the model (10) includes weight and is expected to recover a mean through its zero-order term. Then, the standardized PDF is expressed as:

$$pdf(y) = \sigma_M PDF_M(\sigma_M y) \tag{18}$$

where

$$\sigma_M = \sqrt{\int_{M_S}^{\infty} PDF_M(u)\left(u - \mu_{\psi 0}\right)^2 du} \tag{19}$$

where PDF_M is defined by equation (11) or (16) depending whether the condition (14) is satisfied or not.

The direction of integration from bow or from stern leads to different results, as only FKHS forces are included, no dynamical equilibrium occurs in the section $x = \psi$. This closure error is expected to be eliminated by the scaling procedure (18), but the comprehensive study of this matter has not yet been conducted.

The numerical calculation of the integrals in equations (11) and (16) may be non-trivial for small values of σ_a, since the non-zero values of the integrands may exist only for a small portion of the integration range. The application of adaptive integration scheme (*e.g.* Heath, 2002) helps to resolve the problem.

2.5 Example

Sapsis et al. (2020) describes a large dataset of VBM values produced with Large Amplitude Motion Program (LAMP, e.g. Shin et al. 2003). The volume of the dataset was 5,000 hours of head seas at a forward speed of 10 knots. The simulations were performed for ONR Topsides flared hull, shown in Figure 1 (Bishop et al. 2005) and included Sea State 7 among other conditions.

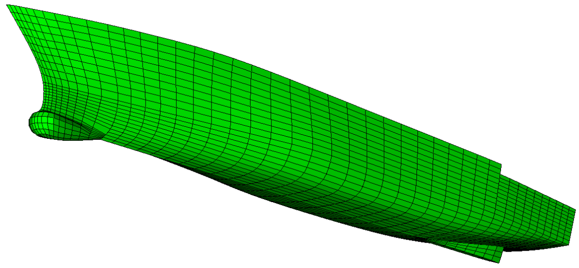

Figure 1: ONR Topsides Flared Hull

Figure 2 shows two standardized histograms from the Sea State 7 simulations and a standardized PDF computed with the approximate volume-based model from formula (11) for case 1 of the semi-analytical PDF. The difference with example from Sapsis et al. (2020) is the accounting for heave. The shape parameter σ_a had to be set to 12 m to achieve the fit, which is almost 2.5 time larger than the pitch only case in Sapsis et al. (2020). The reason for the difference is not known.

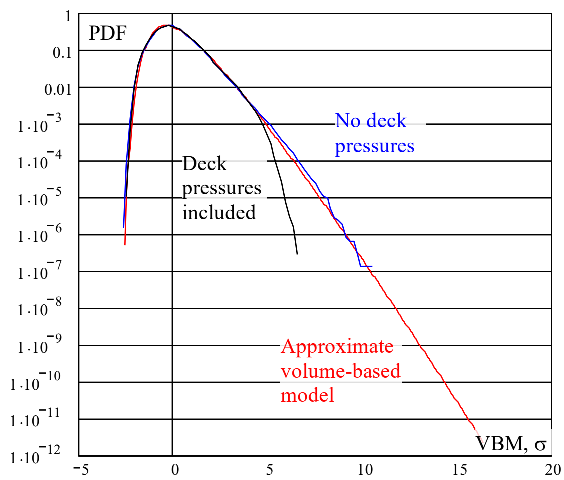

Figure 2: PDF of the approximated volume-based model in comparison with standardized histograms of VBM for SS7.

3. DECK SUBMERGENCE

As expected, the approximate volume-based model has successfully modelled the known asymmetry between sagging and hogging but did not reproduce the influence of the deck submergence. The latter was crudely modelled in simulations by including deck pressures during deck submergence events. This resulted in the appearance of an "inflection" point at the standardized histogram corresponding to about 5 standard deviations of VBM (Sapsis et al. 2020).

As is was shown in the cited reference, the location of the "inflection" point indicates a limit of the applicability of the Weibull fit, and thus has practical interest. The following is a description of ideas how to get there from here.

3.1 Model of Deck Submergence Events

Consider a theoretical situation when a portion of the deck forward of a location x_d is submerged, Figure 3. The distance x_d is measured from a section of interest at location ψ.

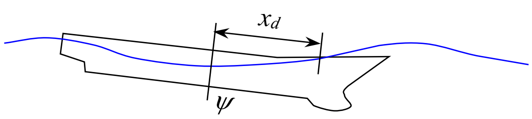

Figure 3: Deck submergence event

The VBM created by the FKHS forces in the section ψ is:

$$M_\psi = \int_\Psi^{\frac{L}{2}} xA(x, d(x))\, dx = \int_\Psi^{x_d} xA(x, d(x))dx + \int_{x_d}^{\frac{L}{2}} xA(x, D)dx \quad (20)$$

where $d(x)$ is a draft at location x while D is a depth (assumed to be constant for the sake of simplicity). The second integral in equation (20) is no longer influenced by the wave surface, it depends only on a volume between x_d and forward perpendicular. It can be pre-computed as a function of x_d:

$$M_D(x_d) = \int_{x_d}^{\frac{L}{2}} xA(x, D)dx \quad (21)$$

The submergence of the deck at a point x_d, on a ship moving in irregular waves can be treated as a problem of the relative motion of a point x_d with respect to the wave surface. At the first expansion, the relative motion can be assumed to follow normal distribution:

$$f(z_d) \sim N(E_{zd}; V_{zd}) \quad (22)$$

where E_{zd} is a mean value and V_{zd} is a variance of the relative motion of the deck point at location x_d.

The deck submergence events at the point x_d can be treated as an exceedance of the freeboard D-d. The probability of deck submergence forward to x_d is then expressed as:

$$P_{x_d}(z_d > D - d) = \int_{D-d}^{\infty} \frac{dz_d}{\sqrt{2\pi V_{zd}}} \exp\left(-\frac{(z_d - E_{zd})^2}{2V_{zd}}\right) = 1 - CDF_N(D - d) \quad (23)$$

3.2 Re-Calculation of Distribution

A mapping between the VBM value (either an instantaneous value or a peak value) and the event of deck submergence are evaluated. A low limit for this mapping exists in terms of x_d position. The second integral in equation (20) equals zero at $x_d = 0.5L$, so no influence of deck submergence event occurs in the VBM value.

The probability that the point $x_d = 0.5L$ is submerged is computed with equation (23) for $x_d = 0.5L$:

$$P_1 = P_{x_d=0.5L}(z_d > D - d) \quad (24)$$

The value of VBM corresponding to this probability is found as a quantile of distribution (11) or (16) for an instantaneous value of VBM and as a quantile of the Weibull distribution for a peak value of VBM:

$$M_{\psi 1} = Q_\psi(P_1) \quad (25)$$

where $M_{\psi 1}$ is the minimum VBM value where deck submergence has its influence.

For an arbitrary point x_d, $\psi < x_d < 0.5L$, the probability of the deck submergence event (is lower than P_1) is computed with equation (23)

$$P_2 = P_{xd}(z_d > D - d)\,;\ \psi < x_d < 0.5L \tag{26}$$

The corresponding value of the VBM without taking into account the influence of the deck submergence event is.

$$M_{\psi 2} = Q_{\psi}(P_2) \tag{27}$$

The FKHS portion of this VBM value can be presented as:

$$\begin{gathered} M_{\psi 2} = M_{\psi 21} + M_{\psi 22} \\ M_{\psi 21} = \int_{\Psi}^{x_d} xA(x, d(x))dx \\ M_{\psi 22} = \int_{x_d}^{\frac{L}{2}} xA(x, d(x))dx \end{gathered} \tag{28}$$

$M_{\psi 22}$, the second component in equation (28), is essentially the VBM value computed for the section located at x_d. Similar to $M_{\psi 2}$, it can be estimated as a quantile of distribution (11) or (16) for an instantaneous value of VBM at a section x_d and as a quantile of the Weibull distribution for a peak value of VBM at a section x_d.

$$M_{xd} = Q_{x_d}(P_2) \tag{29}$$

To correct for the deck submergence influence, the second component in equation (28) has to be substituted by the second integral from equation (20) describing the contribution from the fully submerged portion of the hull:

$$M_{\psi 2}^{*} = M_{\psi 2} - M_{xd} + \int_{x_d}^{\frac{L}{2}} xA(x, D)dx \tag{30}$$

For a sufficiently small probability P_2, the value of M_{xd} is expected to exceed the integral in equation (30), as M_{xd} is a result of extrapolation, while the integral only depends on x_d. As a result, the corrected VBM value, corresponding to the probability level P_2 is smaller than the originally extrapolated estimate. The PDF curves "flexes down," modelling the deck submergence effect observed in Figure 2. A possible scheme is shown in Figure 4.

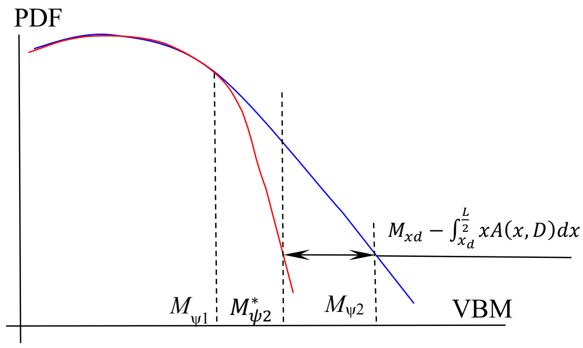

Figure 4: Recalculation of PDF for deck submergence effect

3.3 Alternative Approach

Another approach is to combine the deck submergence effect with the second-order Taylor expansion of the station area at the instantaneous draft. From a step function formulation, the effect of deck submergence is incorporated into the probabilistic analysis. Specifically, a closed form correction can be applied directly to the probability distribution function, derived from the second-order Taylor expansion. Accounting for the dependence of pitch angle only for the first expansion gives:

$$\begin{gathered} m_{\psi 0} = \int_{\Psi}^{\min\left(\frac{x_d}{\theta}, \frac{L}{2}\right)} xA(x, \theta x)\,dx \\ m_{\psi c1} = \int_{\Psi}^{\min\left(\frac{x_d}{\theta}, \frac{L}{2}\right)} xA'(x, \theta x)\cos(k_w x)\,dx \\ m_{\psi s1} = \int_{\Psi}^{\min\left(\frac{x_d}{\theta}, \frac{L}{2}\right)} xA'(x, \theta x)\sin(k_w x)\,dx \\ m_{\psi c2} = \int_{\Psi}^{\min\left(\frac{x_d}{\theta}, \frac{L}{2}\right)} xA''(x, \theta x)\cos(k_w x)\,dx \\ m_{\psi s2} = \int_{\Psi}^{\min\left(\frac{x_d}{\theta}, \frac{L}{2}\right)} xA''(x, \theta x)\sin(k_w x)\,dx \\ m_{\psi cs} = \\ \int_{\Psi}^{\min\left(\frac{x_d}{\theta}, \frac{L}{2}\right)} xA''(x, \theta x)\cos(k_w x)\sin(k_w x)\,dx \end{gathered} \tag{31}$$

The derived model will allow for a physically interpretable representation of the VBM statistics in terms of the hull properties and the wave statistics. However, while in the previous case of no-deck submergence, the above coefficients could be averaged over θ, but averaging is not meaningful in the present context.

A viable approach in this case is to solve the pitch equation in order to obtain the joint statistics between the coefficients $a_c(t), a_s(t)$ and the pitch angle, $\theta(t)$. With this information and an off-line calculation of these coefficients as functions of θ , an extremely efficient computational scheme can be developed that will explicitly account for deck submergence effects.

On the other hand, a simpler approach would ignore the correlation between the wave coefficients $a_c(t), a_s(t)$ and the pitch angle $\theta(t)$. In this case, the computation of these coefficients can be split into two branches/cases:

1) The first branch would be associated with small values of θ, representing the case of no deck submergence. For this regime, the calculation can be identical as in the previous work (Sapsis et al., 2020) where the pdf is averaged over θ.
2) The second branch would be associated with larger values of θ. For this case, the average value of θ can be defined, given that deck submergence has occurred, $\bar{\theta}_s$. For this value, the deck-submergence-relevant coefficients is computed in equation (31) for $\theta = \bar{\theta}_s$.

A random model can then be designed as in equation (10), but with the main difference being that the coefficients will be selected randomly for each sample of amplitude coefficients, either from branch (1) or (2). The probability of selecting one or the other will be defined by equation (24) and the selection will be independent from the wave coefficients. Despite the fact that the correlation between deck submergence and wave coefficients is ignored, the proposed model has the major advantage that it can lead to closed analytical expressions, similar to the no deck submergence case.

Preliminary numerical calculations confirm that the values of the quadratic coefficients for the second branch (i.e. the submergence of deck regime) are significantly smaller compared with first branch. Given this property, it is natural to expect that the tails for the VBM will present an inflection point switching from heavy-tailed (due to the large quadratic coefficients in the no deck submergence regime) to an almost Gaussian behaviour, corresponding to weak quadratic coefficients during deck submergence.

4. SUMMARY AND CONCLUSIONS

Incremental developments are described for an approximate volume-based model for the PDF of VBM, originally derived by Sapsis et al. (2020) and Brown and Pipiras (2020):

- Influence of heave has been added;
- The semi-analytical PDF equation has been extended.

Two ideas for inclusion of deck effects are described:

- Recalculation of the VBM PDF, with similar PDFs, computed for other section and probabilities of deck submergence events
- With a step function formulation, the effect of deck submergence is incorporated into the probabilistic analysis.

The semi-analytical model of VBM PDF holds the promise of a future physics-based tool for the analysis of extreme loads. The next steps are envisioned as follows:

- Development of the approximate volume-based model towards complete internal consistency and elimination of the closure error,
- Development of techniques to fit the model with numerical simulation data,
- Implementation and testing of methods for inclusion of deck submergence effect,
- Statistical validation.

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